

# The Fulling-Davies-Unruh Effect is Mandatory: The Proton's Testimony

George E. A. Matsas\*

Instituto de Física Teórica, Universidade Estadual Paulista  
Rua Pamplona 145, 01405-900, São Paulo, SP  
Brazil

Daniel A. T. Vanzella†

Physics Department, University of Wisconsin-Milwaukee  
1900 E Kenwood Blvd., Milwaukee, WI 53211  
USA

## Abstract

We discuss the *decay of accelerated protons* and illustrate how the Fulling-Davies-Unruh effect is indeed *mandatory* to maintain the consistency of *standard* Quantum Field Theory. The confidence level of the Fulling-Davies-Unruh effect must be the same as that of Quantum Field Theory itself.

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\*matsas@ift.unesp.br

†vanzella@uwm.edu

The vacuum is one of the most exquisite features of Quantum Field Theory (QFT) [1]. Although virtual particles cannot be directly detected, the explanation of a number of effects depend on their “existence”. One of the most outstanding manifestations of the virtual particles can be found in the Casimir effect, which has been tested recently up to a precision of 1% [2]. The Casimir effect is a direct consequence of the fact that the *metallic* plates disturb the *electromagnetic* vacuum between them. If metallic plates can disturb the photon vacuum, the curvature of the spacetime should, in general, disturb all vacua, since gravity couples to all the fields. The Hawking effect is probably the most eloquent example of the particularly relevant rôle which is reserved to the concept of quantum vacuum in strong gravitational fields [3].

In 1976 Unruh [4] found that the Minkowski vacuum, i.e., the quantum state associated with the nonexistence of particles with respect to inertial observers, corresponds to a thermal bath of particles at temperature  $T_{\text{FDU}} = a/2\pi$  ( $\hbar = c = k = 1$ ) to uniformly accelerated observers with proper acceleration  $a = \text{const.}$  This has clarified previous results by Davies [5], and confirmed Fulling’s conclusion that *elementary particles* are observer dependent [6]. Roughly speaking, it can be said that uniformly accelerated observers can see as *real* those particles which inertial observers claim to be *virtual*. As a result, while inertial observers would be frozen at 0 K in the Minkowski vacuum, uniformly accelerated observers would be heated (and possibly burnt) at a temperature proportional to their proper acceleration.

In spite of the fact that the Fulling-Davies-Unruh (FDU) effect can be rigorously derived and extended to nonlinear quantum fields [7] from the general Bisognano and Wichmann’s theorem [8], the technicalities involved and probably its “paradoxical appearance” has kept part of the community quite skeptical up to now (see, e.g., Ref. [9]). Many physicists, thus, have decided to “leave the case to the experiments”. Notwithstanding (what would be considered) a direct manifestation of the FDU effect would require huge accelerations since  $T_{\text{FDU}} = [a/(2.5 \times 10^{22} \text{cm/s}^2)] \text{K}$ . Clearly, macroscopic bodies would not be able to resist such accelerations and, thus, from the very beginning all hopes were placed in elementary particle experiments [10]. Even so, however, paramount technical difficulties have frustrated such endeavors although new and creative proposals have been continuously devised [10]-[14] (see Ref. [15] for a comprehensive list).

Although the observation of direct manifestations of the FDU effect would be sympathetically received, *the acceptance that the FDU effect is mandatory to maintain (the usual) QFT consistent should not depend on it.* Inspired by previous works by Unruh and Wald [16], Higuchi, Sudarsky and one of the authors have used the FDU effect to make sense, in the quantum realm, of the classical result [17]- [18] that *uniformly accelerated charges do not radiate with respect to coaccelerated observers*. Indeed it was shown that *every Minkowski photon emitted by a uniformly accelerated charge (as defined in the inertial frame) corresponds to either the emission to or the absorption from the FDU thermal bath of a zero-energy Rindler photon (as defined in the uniformly accelerated frame)* [19]. (Zero-energy particles are not detectable by physical observers, i.e., with finite proper acceleration.) Perhaps because the concept of *zero-energy* particles might have sounded ethereal to some, the convincing power of the conclusion above did not show itself to be strong enough to reach a consensus in favor of the FDU effect. Here we call attention to the fact that *if the FDU*

effect does not exist, inertial and uniformly accelerated observers would reach incompatible QFT conclusions about the stability of non-inertial protons.

The stability of protons has been used for long time as a test for the standard model of elementary particles. Mounting data fix their lifetime to be much longer than the present age of the universe. This is not so, however, if protons are accelerated rather than freely moving. Recently, the present authors have calculated in the context of *standard QFT* (in inertial frames) the weak-interaction decay rate for uniformly accelerated protons [20]:

$$p^+ \rightarrow n^0 + e_M^+ + \nu_M \quad (1)$$

and shown that in certain astrophysical situations the proton lifetime may be quite short. The energy necessary to render process (1) possible is supplied by the external accelerating agent. (We emphasize, however, that our final conclusion will not depend on the observation of the proton decay itself.) For sake of simplicity, we write the formula for the proton proper lifetime in *1+1 spacetime dimensions* [21] (rather than in 3+1 ones):<sup>1</sup>

$$\tau_{\text{In.Observ.Calcul.}}^{p \rightarrow n}(a) = \frac{2\pi^{3/2} e^{\pi\Delta m/a}}{G_F^2 m_e} \left[ G_{1\,3}^{3\,0} \left( \frac{m_e^2}{a^2} \middle| \begin{array}{c} 1 \\ -1/2, 1/2 + i\Delta m/a, 1/2 - i\Delta m/a \end{array} \right) \right]^{-1}, \quad (2)$$

where  $G_{pq}^{mn}$  is the Meijer function [22],  $a$  is the proton proper acceleration,  $\Delta m \equiv m_n - m_p$  and we have assumed  $m_\nu = 0$ . (Here  $m_p$ ,  $m_n$ ,  $m_e$  and  $m_\nu$  are the rest masses of the proton, neutron, electron and neutrino, respectively.) The value of the effective Fermi constant  $G_F = 9.9 \times 10^{-13}$  is determined from phenomenology. (Because in four dimensions inertial neutrons decay in 887 s, we have chosen this value for the neutron lifetime in two dimensions as well.)

Since a uniformly accelerated proton can be confined in a Rindler wedge which is a globally hyperbolic spacetime possessing a global timelike isometry, the associated uniformly accelerated observers (Rindler observers) must be able to analyze this phenomenon and reobtain the same (scalar) value for the proton lifetime (2) obtained with standard QFT. Notwithstanding, because of energy conservation, Rindler observers would simply claim that protons are precluded from decaying into a neutron through

$$p^+ \rightarrow n^0 + e_R^+ + \nu_R. \quad (3)$$

Hence an extra ingredient must be added, otherwise inertial and Rindler observers would conclude precisely the opposite about the stability of uniformly accelerated protons. This extra ingredient is the FDU thermal bath. According to the Rindler observers, the Minkowski

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<sup>1</sup>This formula was derived, at the tree level, assuming a Fermi-like effective action through which a semi-classical current (describing the barions) is coupled to the fermionic fields (describing the leptons). Eq. (2) is valid under the *no-recoil condition*:  $a \ll m_p$ , used to guaranty that the emitted leptons carry small linear momentum in comparison with  $m_p$ , as measured by an inertial observer instantaneously at rest with the proton.

vacuum is “seen” as a thermal bath with which the proton may interact. As a consequence, new channels are opened:

$$p^+ + e_R^- \rightarrow n^0 + \nu_R \quad (4)$$

$$p^+ + \bar{\nu}_R \rightarrow n^0 + e_R^+ \quad (5)$$

$$p^+ + \bar{\nu}_R + e_R^- \rightarrow n^0, \quad (6)$$

i.e., from the point of view of the Rindler observers, the proton should be transformed into a neutron through the *absorption* of a Rindler electron and/or anti-neutrino from the surrounding thermal bath providing the necessary energy to allow the process to occur. Eventually any energy in excess can be disposed by the emission of a neutrino or a positron (depending on the case). Indeed, by performing an independent QFT calculation in the uniformly accelerated frame, we have obtained the following proper lifetime for the proton, after combining (incoherently) processes (4)-(6) in the presence of the FDU thermal bath:

$$\tau_{\text{Rin.Observ.Calcul.}}^{p \rightarrow n} = \frac{\pi^2 a e^{\pi \Delta m/a}}{G_F^2 m_e} \left[ \int_{-\infty}^{+\infty} d\omega_R \frac{K_{i\omega_R/a+1/2}(m_e/a) K_{i\omega_R/a-1/2}(m_e/a)}{\cosh[\pi(\omega_R - \Delta m)/a]} \right]^{-1}. \quad (7)$$

Although Eqs. (2) and (7) appear to be quite different, we have shown numerically that they coincide up to basically the machine-precision limit [23]:

$$\Delta^{-1} \int_{\Delta} dx [(\tau_{\text{Rin.Observ.Calcul.}}^{p \rightarrow n} - \tau_{\text{In.Observ.Calcul.}}^{p \rightarrow n}) / \tau_{\text{Rin.Observ.Calcul.}}^{p \rightarrow n}]^2 \sim 10^{-16},$$

where  $x \equiv \log_{10}(a/1\text{MeV})$ .

The FDU effect is not only very important for its own right but is also useful as a guide in the investigation of some unresolved questions of the Hawking effect as, e.g., the rôle played by the transplackian frequencies (see Ref. [24] and references therein). The FDU effect must be seen as being as necessary to QFT as the non-inertial forces (centrifugal and Coriolis ones) are to Mechanics since both are required to maintain successfully tested theories consistent when analyzed in non-inertial frames. The description of the decay of non-inertial protons in the uniformly accelerated frame *in the absence of the FDU thermal bath* is the *challenge* which the present Essay poses for those who are still doubtful about this effect.

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